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Stochastic Model for Nonisothermal Droplet-Laden Turbulent Flows

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A stochastic model is proposed for the prediction of velocity and temperature fluctuations in turbulent flows laden with nonevaporating droplets. The model is based on a first-order time series analysis, addresses the anisotropy of turbulence, and adequately takes into account the temporal correlations. The gravity effect is not considered in this study. The performance of the model is assessed by conducting simulations of droplet-laden homogeneous shear flows. The predictions of the stochastic model are compared with the results from a model derived using a kinetic approach and the data from a direct numerical simulation study. Excellent agreements are observed for various velocity and temperature statistics.

Nomenclature

$\boldsymbol{A}$	=	(u, v, w, θ) vector of carrier phase fluctuations
$\boldsymbol{C}$	=	fluid temporal correlations matrix
d_p	=	droplet diameter
k	=	carrier-phase turbulence kinetic energy
G_u	=	$\mathrm{d}U_1/\mathrm{d}y$ mean velocity gradient of the carrier phase
G_θ	=	$\mathrm{d}T/\mathrm{d}y$ mean temperature gradient of the carrier phase
T	=	mean temperature of the carrier phase
$\tilde{T}$	=	instantaneous temperature of the carrier phase
$\tilde{T}_p$	=	instantaneous temperature of the droplet
t	=	time
$\boldsymbol{U}$	=	(U_1, U_2, U_3) mean velocity of the carrier phase
u	=	fluctuating velocity of the carrier phase in x_1 direction
$\boldsymbol{u}$	=	fluctuating velocity of the carrier phase
$\tilde{\boldsymbol{u}}$	=	instantaneous velocity of the carrier phase
$\tilde{\boldsymbol{u}}_p$	=	(v_1, v_2, v_3) instantaneous velocity of the droplet
v	=	fluctuating velocity of the carrier phase in x_2 direction
w	=	fluctuating velocity of the carrier phase in x_3 direction
x_i	=	Eulerian spatial coordinates
$\tilde{\boldsymbol{x}}_p$	=	droplet instantaneous position
α	=	$\mathrm{d}U_1/\mathrm{d}y$ mean velocity gradient
ϵ	=	viscous dissipation rate
θ	=	fluctuating temperature of the carrier phase
θ_p	=	fluctuating temperature of the droplet
τ_p	=	droplet time constant

Subscripts

p	=	droplet properties
t	=	time

Superscript

T	=	transpose of a matrix

Introduction

TURBULENT flows laden with a dispersed phase of solid particles or liquid droplets have been extensively investigated for several decades, due primarily to their broad ranges of applications. The main difficulty in providing a mathematical description for prediction of these flows arises from the presence of turbulence that is characterized by a large variation in length scales and timescales. A variety of approaches have been adopted for description of two-phase turbulent flows; however, it appears that in practical situations the stochastic approach has been more successful than other methods. The literature[1–8] is rich with previous contributions on development and application of various stochastic models; however, the majority of the work has been focused on isothermal flows, where only velocity fluctuations are considered. Because heat transfer occurs naturally in many industrial processes, for example, spray combustion in engines, combustion of coal particles in large furnaces, spray drying in the food industry, pneumatic transport, spray forming in manufacturing and material processing, gas-particle combustion flow in a solid propellant rocket system, etc., the temperature fluctuations must be taken into account. The main objective of this work is to develop a stochastic model for nonisothermal flows, while accounting for correlations among various components of the velocity field and the temperature field.

As already mentioned, the number of previous works that consistently consider the effect of turbulence on temperature fluctuations is limited. Of direct relevance to the work presented in this paper is a recent work by Moissette et al.,[9] where a model is constructed by implementing first-order autoregressive processes for both velocity and temperature fluctuations of the particles. This model also accommodates for anisotropy effects of turbulence. An important aspect of the model is the inclusion of the correlations between temperature and velocity fluctuations in a systematic manner within the framework of autoregressive processes. The model has been implemented to simulate solid particles dispersed in a homogenous shear turbulence, and good agreements with analytical results of Zaichik[10] have been found.

In the model of Moissette et al.,[9] the turbulent dispersion and temperature fluctuations are treated separately. The model for the velocity fluctuations has no contribution from the temperature field, and the correlation between temperature and velocity is considered only to obtain temperature fluctuations. The model also does not account for temporal variation of correlations; therefore, this model may not work as accurately in a temporally developing turbulence. In this paper, we present a more general stochastic model for the prediction of temperature fluctuations, that consistently addresses these issues. The performance of the model is assessed by comparisons with previously published data.

Problem Formulation

We consider dispersion of nonevaporating droplets (or solid particles) in a carrier turbulent flow. The carrier-phase motion is governed by the Navier–Stokes equations, which are not repeated here

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because we are exclusively concerned with the treatment of the dispersed phase. The droplets are assumed to be spheres and exhibit an empirically corrected Stokesian drag force. The translational motion is the only motion considered for the droplets, and their rotation is neglected. The density of the droplets is considered to be constant and much larger than that of the fluid so that only the inertia and the drag force are significant to the droplet dynamics. Gravity effects are not considered in this study. The droplets are assumed lumped and each droplet is at a uniform temperature. In addition, both droplet–droplet interaction and heat transfer due to radiation are neglected because a small volume fraction is assumed for droplets. This, however, does not limit the present model to one-way coupling. The effects of the droplets on the turbulence may be included in the Eulerian equations for the carrier gas as source/sink terms, and this procedure does not affect the stochastic model for the droplets.

Each droplet is tracked individually in a Lagrangian frame, and its instantaneous position, velocity, and temperature are denoted by $\tilde{\boldsymbol{x}}_p$, $\tilde{\boldsymbol{u}}_p = (v_1, v_2, v_3)$, and $\tilde{T}_p$, respectively. With this nomenclature, the nondimensional Lagrangian equations describing the droplet dynamics and heat transfer are[11]

$$\frac{\mathrm{d}\tilde{\boldsymbol{x}}_p}{\mathrm{d}t} = \tilde{\boldsymbol{u}}_p \tag{1}$$

$$\frac{\mathrm{d}\tilde{\boldsymbol{u}}_p}{\mathrm{d}t} = \frac{f_1}{\tau_p}(\tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}_p) \tag{2}$$

$$\frac{\mathrm{d}\tilde{T}_p}{\mathrm{d}t} = \frac{f_2}{\tau_p}(\tilde{T} - \tilde{T}_p) \tag{3}$$

where $\tilde{\boldsymbol{u}}$ and $\tilde{T}$ are the fluid velocity vector and temperature at the droplet location, respectively. All of the variables are normalized by reference length L_f, density ρ_f, velocity U_f, and temperature T_f scales. Consequently, reference Reynolds and Prandtl numbers are defined as $Re_f = \rho_f U_f L_f/\mu$ and $Pr = C_p\mu/\kappa$, respectively, where μ, κ, and C_p are the viscosity, the thermal conductivity, and the specific heat of the fluid, respectively.

In Eqs. (2) and (3), the nondimensional droplet time constant is $\tau_p = Re_f\rho_p d_p^2/18$, where d_p and ρ_p are the droplet diameter and density, respectively. The function $f_1 = 1 + 0.15Re_p^{0.687}$ in Eq. (2) represents an empirical correction to the Stokes drag due to droplet Reynolds numbers of order unity and larger (see Ref. 12) and is valid for nondeformable spherical droplet and for droplet Reynolds numbers $Re_p = Re_f\rho d_p|\tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}_p| \leq 1000$, where ρ is the fluid density at the droplet position. The factor $f_2 = Nu/3Pr\sigma$ represents a correlation for the convective heat transfer coefficient based on an empirically corrected Nusselt number (see Ref. 13), $Nu = 2 + 0.6Re_p^{0.5}Pr^{0.33}$, where σ is the ratio of the droplet specific heat and the fluid specific heat.

Stochastic Model

To determine the droplet trajectories, the instantaneous velocity of the fluid at the droplet location is needed. In turbulent flows, although the Navier–Stokes equations are deterministic, it has been proven that chaos can arise from a nonlinear deterministic context and turbulence can be characterized as a stochastic process. There are several methods to describe this stochastic process. One is to assume the droplet motion as a Possion-like process with a known probability density function. The eddy-interaction model of Gosman and Ioannides[1] is based on this method. Another method is to describe the process by stochastic differential equations (SDEs).[14,15] An interesting discussion on the application of this method for modeling of gas–solid turbulent flows is provided by Pozorski and Minier.[8] The SDE methodology is still in the development stages and has yet not been widely used for most engineering applications.[16] The third method is via the application of the method of time series analysis, which assumes the process is a Markovian chain. In this paper, we will use this method to construct a new stochastic model, and the procedure is explained in the following paragraphs.

The instantaneous velocity and temperature of the carrier phase at the droplet location can be decomposed into mean $\boldsymbol{U} \equiv (U_1, U_2, U_3)$ and T and fluctuation $\boldsymbol{u} \equiv (u_1, u_2, u_3) \equiv (u, v, w)$ and θ, respectively. The mean velocity and temperature of the fluid can be calculated by various single-point statistical models such as those devised in the Reynolds-average Navier–Stokes framework. The computation of the mean values is, therefore, not discussed here, instead we focus on the calculation of the fluctuating quantities. A stochastic modeling approach is adopted for this purpose.

The stochastic approach is based on the work of Zhou and Leschziner,[17] who propose a model for velocity fluctuations within the framework of a first-order time series analysis.[18] Here, we extend this approach to include the modeling of the temperature field. We start by describing the fluctuating quantities at time t in terms of their corresponding values at a previous time $t - \delta t$:

$$u_t = \beta_{uu}u_{t-\delta t} + \beta_{uv}v_{t-\delta t} + \beta_{uw}w_{t-\delta t} + \beta_{u\theta}\theta_{t-\delta t} + d_{tu} \tag{4}$$

$$v_t = \beta_{vu}u_{t-\delta t} + \beta_{vv}v_{t-\delta t} + \beta_{vw}w_{t-\delta t} + \beta_{v\theta}\theta_{t-\delta t} + d_{tv} \tag{5}$$

$$w_t = \beta_{wu}u_{t-\delta t} + \beta_{wv}v_{t-\delta t} + \beta_{ww}w_{t-\delta t} + \beta_{w\theta}\theta_{t-\delta t} + d_{tw} \tag{6}$$

$$\theta_t = \beta_{\theta u}u_{t-\delta t} + \beta_{\theta v}v_{t-\delta t} + \beta_{\theta w}w_{t-\delta t} + \beta_{\theta\theta}\theta_{t-\delta t} + d_{t\theta} \tag{7}$$

which may be recast in matrix form as

$$\boldsymbol{A}_t = \boldsymbol{\beta} \cdot \boldsymbol{A}_{t-\delta t} + \boldsymbol{d}_t \tag{8}$$

$$\boldsymbol{A}_t = \begin{pmatrix} u_t \\ v_t \\ w_t \\ \theta_t \end{pmatrix}, \qquad \boldsymbol{\beta} = \begin{pmatrix} \beta_{uu} & \beta_{uv} & \beta_{uw} & \beta_{u\theta} \\ \beta_{vu} & \beta_{vv} & \beta_{vw} & \beta_{v\theta} \\ \beta_{wu} & \beta_{wv} & \beta_{ww} & \beta_{w\theta} \\ \beta_{\theta u} & \beta_{\theta v} & \beta_{\theta w} & \beta_{\theta\theta} \end{pmatrix}$$

$$\boldsymbol{d}_t = \begin{pmatrix} d_{tu} \\ d_{tv} \\ d_{tw} \\ d_{t\theta} \end{pmatrix} \tag{9}$$

The main task is now to derive relations for $\boldsymbol{\beta}$ and $\boldsymbol{d}_t$ so that the fluctuations $\boldsymbol{A}_t$, at time t, can be determined from the known fluctuations $\boldsymbol{A}_{t-\delta t}$, at time $t - \delta t$. The derivation of relations for $\boldsymbol{\beta}$ and $\boldsymbol{d}_t$ is described next.

We first multiply both sides of Eq. (8) by $\boldsymbol{A}_{t-\delta t}^T$ and take the expectation of the resulting equation to obtain

$$E\big[\boldsymbol{A}_t \cdot \boldsymbol{A}_{t-\delta t}^T\big] = E\big[(\boldsymbol{\beta} \cdot \boldsymbol{A}_{t-\delta t} + \boldsymbol{d}_t) \cdot \boldsymbol{A}_{t-\delta t}^T\big] \tag{10}$$

where the superscript T denotes the transpose of a matrix. The left-hand side (LHS) of Eq. (10) is the temporal correlation matrix $\boldsymbol{C}$ defined as

$$\boldsymbol{C} = \begin{pmatrix} \overline{u_t u_{t-\delta t}} & \overline{u_t v_{t-\delta t}} & \overline{u_t w_{t-\delta t}} & \overline{u_t \theta_{t-\delta t}} \\ \overline{v_t u_{t-\delta t}} & \overline{v_t v_{t-\delta t}} & \overline{v_t w_{t-\delta t}} & \overline{v_t \theta_{t-\delta t}} \\ \overline{w_t u_{t-\delta t}} & \overline{w_t v_{t-\delta t}} & \overline{w_t w_{t-\delta t}} & \overline{w_t \theta_{t-\delta t}} \\ \overline{\theta_t u_{t-\delta t}} & \overline{\theta_t v_{t-\delta t}} & \overline{\theta_t w_{t-\delta t}} & \overline{\theta_t \theta_{t-\delta t}} \end{pmatrix} \tag{11}$$

which may be described in terms of correlation functions $R_{ij}(\delta t)$ and various standard deviations of the fluid fluctuating velocity components and temperature at times t and $(t - \delta t)$ as

$$
\boldsymbol{C} = \begin{pmatrix}
R_{uu}\sqrt{u_t^2}\sqrt{u_{t-\delta t}^2} & R_{uv}\sqrt{u_t^2}\sqrt{v_{t-\delta t}^2} & R_{uw}\sqrt{u_t^2}\sqrt{w_{t-\delta t}^2} & R_{u\theta}\sqrt{u_t^2}\sqrt{\theta_{t-\delta t}^2} \\
R_{vu}\sqrt{v_t^2}\sqrt{u_{t-\delta t}^2} & R_{vv}\sqrt{v_t^2}\sqrt{v_{t-\delta t}^2} & R_{vw}\sqrt{v_t^2}\sqrt{w_{t-\delta t}^2} & R_{v\theta}\sqrt{v_t^2}\sqrt{\theta_{t-\delta t}^2} \\
R_{wu}\sqrt{w_t^2}\sqrt{u_{t-\delta t}^2} & R_{wv}\sqrt{w_t^2}\sqrt{v_{t-\delta t}^2} & R_{ww}\sqrt{w_t^2}\sqrt{w_{t-\delta t}^2} & R_{w\theta}\sqrt{w_t^2}\sqrt{\theta_{t-\delta t}^2} \\
R_{\theta u}\sqrt{\theta_t^2}\sqrt{u_{t-\delta t}^2} & R_{\theta v}\sqrt{\theta_t^2}\sqrt{v_{t-\delta t}^2} & R_{\theta w}\sqrt{\theta_t^2}\sqrt{w_{t-\delta t}^2} & R_{\theta\theta}\sqrt{\theta_t^2}\sqrt{\theta_{t-\delta t}^2}
\end{pmatrix} \tag{12}
$$

The right-hand side (RHS) of Eq. (10) can be expressed as

$$\boldsymbol{\beta}\cdot E\left[\boldsymbol{A}_{t-\delta t}\cdot\boldsymbol{A}_{t-\delta t}^T\right] + E\left[\boldsymbol{d}_t\cdot\boldsymbol{A}_{t-\delta t}^T\right] = \boldsymbol{\beta}\cdot\operatorname{cov}\left(\boldsymbol{A}_{t-\delta t},\boldsymbol{A}_{t-\delta t}^T\right) \tag{13}$$

where $\boldsymbol{d}_t$ and $\boldsymbol{A}_{t-\delta t}^T$ are two independent random variables based on the theory of time series analysis and

$$\operatorname{cov}\left(\boldsymbol{A}_{t-\delta t},\boldsymbol{A}_{t-\delta t}^T\right) = \begin{pmatrix}
\overline{uu} & \overline{uv} & \overline{uw} & \overline{u\theta} \\
\overline{vu} & \overline{vv} & \overline{vw} & \overline{v\theta} \\
\overline{wu} & \overline{wv} & \overline{ww} & \overline{w\theta} \\
\overline{\theta u} & \overline{\theta v} & \overline{\theta w} & \overline{\theta\theta}
\end{pmatrix}_{t-\delta t} \tag{14}$$

From Eqs. (10), (11), and (13), we can write $\boldsymbol{C} = \boldsymbol{\beta}\cdot\operatorname{cov}(\boldsymbol{A}_{t-\delta t}, \boldsymbol{A}_{t-\delta t}^T)$, or

$$\boldsymbol{\beta} = \boldsymbol{C}\cdot\operatorname{cov}^{-1}\left(\boldsymbol{A}_{t-\delta t},\boldsymbol{A}_{t-\delta t}^T\right) \tag{15}$$

Next, we focus on deriving an expression for $\boldsymbol{d}_t$ by writing Eq. (8) as

$$\boldsymbol{d}_t = \boldsymbol{A}_t - \boldsymbol{\beta}\cdot\boldsymbol{A}_{t-\delta t} \tag{16}$$

Multiplying both sides by their respective transpose, and then taking the expectations, leads to

$$E\left[\boldsymbol{d}_t\cdot\boldsymbol{d}_t^T\right] = E\left[(\boldsymbol{A}_t - \boldsymbol{\beta}\cdot\boldsymbol{A}_{t-\delta t})\cdot\left(\boldsymbol{A}_t^T - \boldsymbol{A}_{t-\delta t}^T\cdot\boldsymbol{\beta}^T\right)\right] \tag{17}$$

whose LHS is simply $\operatorname{cov}(\boldsymbol{d}_t, \boldsymbol{d}_t^T)$ and its RHS can be simplified using Eq. (15), to obtain

$$\operatorname{cov}\left(\boldsymbol{d}_t,\boldsymbol{d}_t^T\right) = \operatorname{cov}\left(\boldsymbol{A}_t,\boldsymbol{A}_t^T\right) - \boldsymbol{\beta}\cdot\boldsymbol{C}^T \tag{18}$$

To proceed, we write

$$\boldsymbol{d}_t = \boldsymbol{B}\cdot\boldsymbol{Z} \tag{19}$$

where $\boldsymbol{B}$ is a matrix, which needs to be determined, and $\boldsymbol{Z}$ is a random vector, each component of which is independent and sampled from a standard normal distribution with a mean of zero and a variance of unity. Multiplying both sides of Eq. (19) by their transpose, and taking expectations, we obtain

$$E\left[\boldsymbol{d}_t\cdot\boldsymbol{d}_t^T\right] = E\left[\boldsymbol{B}\cdot\boldsymbol{Z}\cdot(\boldsymbol{B}\cdot\boldsymbol{Z})^T\right] \tag{20}$$

which after some simplifications leads to

$$\boldsymbol{B}\cdot\boldsymbol{B}^T = \operatorname{cov}\left(\boldsymbol{d}_t,\boldsymbol{d}_t^T\right) \tag{21}$$

From Eq. (21), $\boldsymbol{B}$ can be determined by using Cholesky factorization of matrix $\operatorname{cov}(\boldsymbol{d}_t, \boldsymbol{d}_t^T)$ because $\operatorname{cov}(\boldsymbol{d}_t, \boldsymbol{d}_t^T)$ is a symmetric positive definite matrix. This completes the determination of $\boldsymbol{\beta}$ and $\boldsymbol{d}_t$ such that fluctuations at time t can be determined from fluctuations at time $t-\delta t$ using Eq. (8).

Model Assessment

To assess the performance of the stochastic model, in this section we consider simulations of a homogeneous shear turbulent flow laden with nonevaporating droplets. As shown in Fig. 1, this flow is characterized by a mean velocity gradient imposed in the cross-stream direction. The magnitude of the velocity gradient, dU_1/dx_2, is constant in time, where indices 1 and 2 denote the streamwise and cross-stream directions, respectively. In addition, to assess the statistics related to the temperature field, a constant mean temperature gradient, dT/dx_2, is also imposed on the flow. The carrier phase is incompressible, and the volume fraction of the droplets is low enough to assume one-way coupling. Two sets of comparisons are provided next.

Equilibrium Homogeneous Shear Turbulence

First, we conduct simulations to compare flow statistics at equilibrium stage of the homogenous shear flow development. At this stage, the values of the normalized statistics do not change with time; thus, the flow is considered at a quasisteady state. The results of our stochastic simulations (STH) in this part are compared with the results of a statistical model developed in the probability density function (PDF) framework by Zaichik[10] and later rederived by Pandya and Mashayek.[19] For convenience, in this part of our assessment we use the same notation as introduced by Zaichik.[10]

The simulations are conducted by injecting a total of 20,000 droplets at $x(\equiv x_1) = 0$ and randomly distributing them over the region $0 \le y(\equiv x_2) \le 1$. The initial velocity and temperature of the droplets are assumed to be the same as those of their surrounding fluid elements. To proceed with the calculations, first a time-step size δt is assigned. Then Eqs. (1–3) are integrated to update the properties of the droplets. To integrate these equations, the instantaneous properties of the fluid phase at the droplet location are needed and can be calculated in two parts, namely, mean and fluctuation. The mean value is calculated from the given velocity and temperature gradients (discussed later) and the fluctuation is obtained by solving Eq. (8). After Eqs. (1–3) are integrated, the droplets are moved to their new locations, and the statistics are calculated by ensemble averaging over all of the droplets. To advance in time, the described procedure is repeated.

Homogeneous flows are, by nature, unbounded, and thus, the results would not depend on the specific values used for the mean values of velocity and temperature, rather to the gradient of these variables. To configure a homogeneous shear flow with a mean velocity gradient of $G_u = dU_1/dy = 20$, we use $U_1 = 1$ and 21 at $y = 0$ and 1, respectively. As the results of our simulation in Fig. 2 indicate, this leads to a droplet distribution that becomes more and more skewed in time. The dashed lines in Fig. 2 represent the motion due to the assigned mean velocities, and the turbulence dispersion of the droplets about these lines is clearly demonstrated by plotting the instantaneous locations of all of the droplets in the computational domain at various times. Further details of the numerical simulations and the choices of other parameters are provided later.

The integral time scales for the velocity, T_u, and temperature, T_θ, of the fluid seen by the droplets are assumed to be constant and equal, that is, $T_u = T_\theta = T^*$. The velocity gradient parameter $S_u = T_u G_u$ is assumed to be equal to the temperature gradient parameter $S_\theta = (T_\theta\sqrt{k}/\sqrt{\overline{\theta^2}})G_\theta$, where $G_\theta = dT/dy$. The dimensionless time parameters Ω_u and Ω_θ are defined as $\Omega_u = \tau_p/T_u$ and

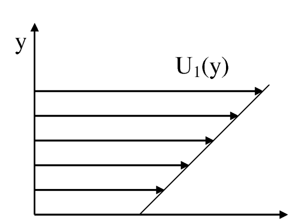

Fig. 1 Schematic of homogeneous shear flow configuration.

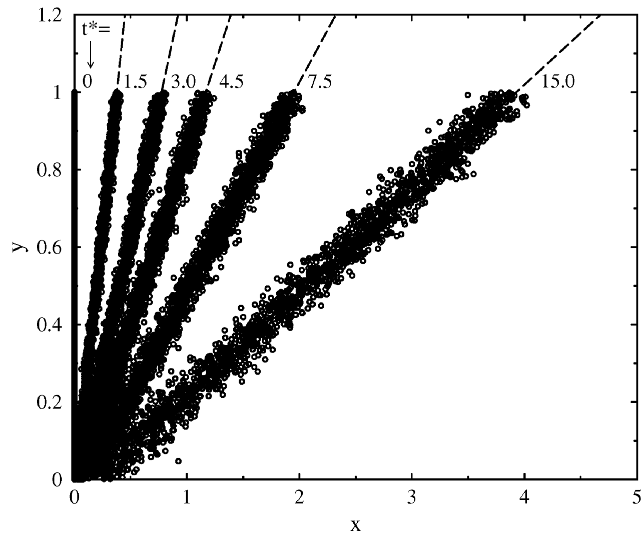

Fig. 2 Droplet distribution at different normalized times $t^* = t/T^*$.

$\Omega_\theta = \tau_{p\theta}/T_\theta$, where $\tau_{p\theta} = \tau_p/f_2$. For comparison purposes, only cases with $\Omega_u = \Omega_\theta$ are investigated, and the correction factors in f_1 and f_2, introduced in Eqs. (2) and (3) due to large droplet Reynolds numbers, are neglected as was the case in the PDF model calculations. The following correlations are used from the experimental data of Tavoularis and Corrsin[20] for the homogeneous shear turbulence:

$$\overline{u^2}/k = 1.07, \qquad \overline{v^2}/k = 0.37, \qquad \overline{w^2}/k = 0.56 \tag{22}$$

$$\overline{uv}\Big/\sqrt{\overline{u^2}}\sqrt{\overline{v^2}} = -0.45, \qquad \overline{u\theta}\Big/\sqrt{\overline{u^2}}\sqrt{\overline{\theta^2}} = 0.59$$

$$\overline{v\theta}\Big/\sqrt{\overline{v^2}}\sqrt{\overline{\theta^2}} = -0.45 \tag{23}$$

where k is the fluid turbulence kinetic energy. These values provide all of the information needed for the calculation of $\mathrm{cov}(\boldsymbol{A}_{t-\delta t}, \boldsymbol{A}_{t-\delta t}^T)$ and $\mathrm{cov}(\boldsymbol{A}_t, \boldsymbol{A}_t^T)$ that lead to the calculation of $\boldsymbol{\beta}$ in Eq. (15) and $\mathrm{cov}(\boldsymbol{d}_t, \boldsymbol{d}_t^T)$ in Eq. (18). The correlation functions are assumed in the following exponential form:

$$R_{\alpha\beta}(\delta t) = \left(\overline{\alpha\beta}\Big/\sqrt{\overline{\alpha^2}}\sqrt{\overline{\beta^2}}\right)\exp(-\delta t/T_{\alpha\beta}) \tag{24}$$

where α and β represent u, v, w, or θ. In the simulations, $T_{\alpha\beta} = T^*$ is assumed. When Eq. (24) is used, the matrix $\boldsymbol{C}$ in Eq. (12) can be calculated. The exponential form for the correlation function has been justified from experiment[21] and also used by Taylor.[22]

For comparison with the PDF model, simulations are conducted for different values of $\Omega_u = \Omega_\theta$ and S_u. For these simulations, the droplet relaxation times τ_p and $\tau_{p\theta}$ are constant and equal. The time step is chosen as 1/10 of the minimum of τ_p and T^*. Because the flow is homogeneous, various statistics are calculated by ensemble averaging over all of the droplets present in the domain. Figure 3 shows comparisons for $\langle v_1^2\rangle$, $\langle v_1 v_2\rangle$, $\langle v_1\theta_p\rangle$, $\langle v_2\theta_p\rangle$, and $\langle\theta_p^2\rangle$ normalized by their corresponding fluid statistics. It is observed that the stochastic simulations produce nearly identical results to PDF model predictions for all values of Ω_u and S_u.

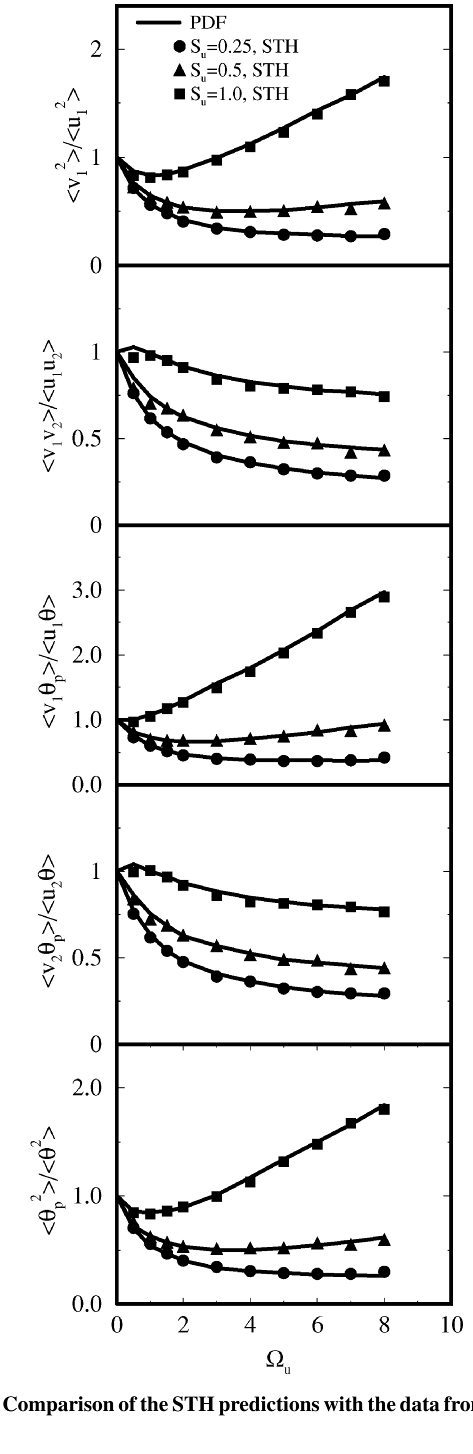

Fig. 3 Comparison of the STH predictions with the data from the PDF model.

Temporally Developing Homogeneous Shear Turbulence

Next, we conduct simulations to compare with direct numerical simulation (DNS) data of Shotorban et al.[23] This comparison provides a means to assess the performance of the stochastic model in a transient flow. Again a homogeneous shear flow is considered with constant mean velocity and mean temperature gradients. The details of the simulations are described by Shotorban et al.[23] and will not be repeated here. Very briefly, a large number of droplets are randomly distributed in an initially isotropic carrier phase with the same velocity and temperature as those of their surrounding fluid elements. The trajectories, velocities, and temperatures of these droplets are then calculated using their corresponding fluid variables from the DNS of the carrier phase. Because the initial flowfield is isotropic, there is no production for the turbulence kinetic energy at $t = 0$ and the turbulence experiences an initial decay. In time, the presence of the normal Reynolds stress components contributes to the production of the shear stress, which in turn results in a production component for the turbulence kinetic energy. Consequently, the turbulence starts to grow after an initial decaying period. The DNS results indicate that the turbulence kinetic energy of the droplets also follows a similar decay and growth in time.

Our stochastic simulations are conducted using similar initial conditions as those used in DNS. Furthermore, the fluid statistics are directly taken from the DNS results, and the comparison is for the statistics of the droplets only. To implement the model, various integral timescales in Eq. (24) for velocity–temperature and

Table 1 DNS calculated values of $T_{\alpha\beta}$ used in conjunction with the correlation functions in Eq. (24)

Timescale	Value
$T_{\theta\theta}$	0.526
$T_{u\theta}$	0.625
$T_{v\theta}$	0.425
$T_{\theta u}$	0.5
$T_{\theta v}$	0.625

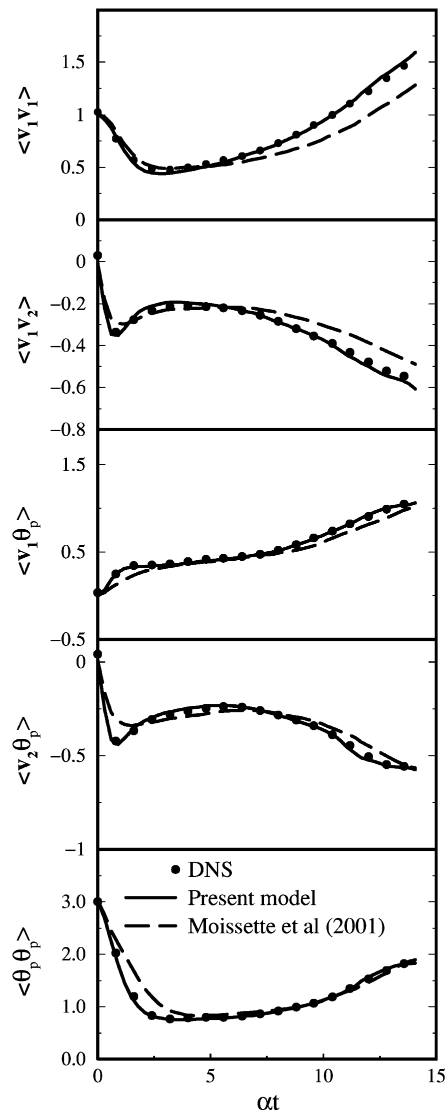

Fig. 4 Comparison of the present stochastic model predictions with DNS data and the results obtained using Moissette et al.[9] model.

temperature–temperature correlations have been evaluated using the DNS data. The nonzero values of these timescales are summarized in Table 1. More details on the evaluation of these timescales may be found by Pandya and Mashayek.[19] For velocity–velocity correlations, $T_{\alpha\beta} = 0.482k/\epsilon$ have been used, where ϵ is the dissipation rate of the fluid turbulence kinetic energy and is taken directly from the DNS results. All of the correlations needed in $\mathrm{cov}(\boldsymbol{A}_{t-\delta t}, \boldsymbol{A}_{t-\delta t}^T)$ and $\mathrm{cov}(\boldsymbol{A}_t, \boldsymbol{A}_t^T)$ calculation are provided via DNS data.

For comparisons, we consider a case with one-way coupling and droplet time constant of $\tau_p = 0.3$. The droplets are injected and tracked in stochastic simulations following exactly the same procedure as that described in the preceding subsection. Figure 4 shows the comparison between the model predictions and the DNS data for temporal variations of $\langle v_1 v_1\rangle$, $\langle v_1 v_2\rangle$, $\langle v_1 \theta_p\rangle$, $\langle v_2 \theta_p\rangle$, and $\langle \theta_p \theta_p\rangle$. In Fig. 4, the time axis has been normalized using the mean velocity gradient magnitude $\alpha = \mathrm{d}U_1/\mathrm{d}y$. All of the variables used in these simulations are nondimensionalized using the same reference scales as those implemented in the DNS formulation. In this nondimensional form the magnitudes of mean velocity and mean temperature gradients are the same, that is, $\mathrm{d}U_1/\mathrm{d}y = \mathrm{d}T/\mathrm{d}y = 2$. The comparisons in Fig. 4 show excellent agreements with the DNS results. Although this clearly indicates the ability of the stochastic model in generating the statistics of the dispersed phase, the challenge of reproducing the fluid statistics still remains open for further research.

To demonstrate the improvement gained by incorporating the temporal correlations, we have recalculated the droplet statistics by using the Moissette et al. model.[9] The results are plotted in Fig. 4 and compared to the new model results and the DNS data. It is clear that the previous model by Moissette et al. shows some deviations from the DNS data for the temporally developing turbulent flow. This is primarily because the model of Moissette et al. does not consider the temporal variation of the correlation function, which demonstrates the important role of the temporal correlation of turbulence in the construction of stochastic models.

Conclusions

A stochastic model has been proposed, within the Lagrangian framework, for the prediction of velocity and temperature fluctuations in turbulent flows laden with nonevaporating droplets. This model is based on a first-order time series analysis and addresses the anisotropy of turbulence. The model also adequately takes into account the temporal correlations of turbulence. In its final form, the droplet stochastic equation contains one random term and one correlation term. In the correlation term, there are provisions for inclusion of both directional and temporal correlations. For the temporal correlation function, an exponential form is assumed.

For a preliminary assessment of the model performance, simulations of homogeneous shear flows are considered, and two comparison studies are conducted. In the first study, we compare the model predictions for statistics obtained for an equilibrium homogeneous shear flow with the predictions from a mathematical model based on a PDF modeling approach. In this comparison, the effects of the shear rate and the droplet size on various statistics are investigated. In the second study, we consider the results generated by DNS and conduct comparisons for temporal evolution of various statistical properties of the dispersed phase. In these comparisons, uniform mean gradients are considered for both velocity and temperature of the carrier phase. To calculate the droplet statistics, ensemble averaging is performed over a large number of droplet trajectories. Various correlations among velocity components and temperature of the droplet phase are calculated and presented. The results are in perfect agreement with the analytical solution for the equilibrium shear flow and with the DNS results for the temporally developing shear flow.

The modeling strategy adopted here is well suited for inclusion of other scalars such as mass fractions of various species that can be used for the description of two-phase combustion systems. This will be considered in our future works.

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